

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

Fox and Bajari
(2013)

References

Matching

Paul Schrimpf

UBC
Economics 565

March 25, 2021

References

- Brief review: Fox (2009)
- Longer review: Graham (2011)
- Extensive notes: Galichon (2011)
- Identification: Fox (2010b), Galichon and Salanié (2010), and many of the papers below
- Applications
 - Marriage: Choo and Siow (2006) , Galichon and Salanié (2010)
 - Mergers: Uetake and Watanabe (2012), Park (2012), Oktay Akkus and Hortaçsu (2012)
 - Venture capital: Sørensen (2007)
 - Downstream - upstream firms: Fox (2018)
 - Medical residents: Agarwal (2012)

- 1 Introduction
- 2 Sørensen (2007)
- 3 Fox (2018)
- 4 Fox and Bajari (2013)

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

Fox and Bajari
(2013)

References

Section 1

Introduction

Introduction

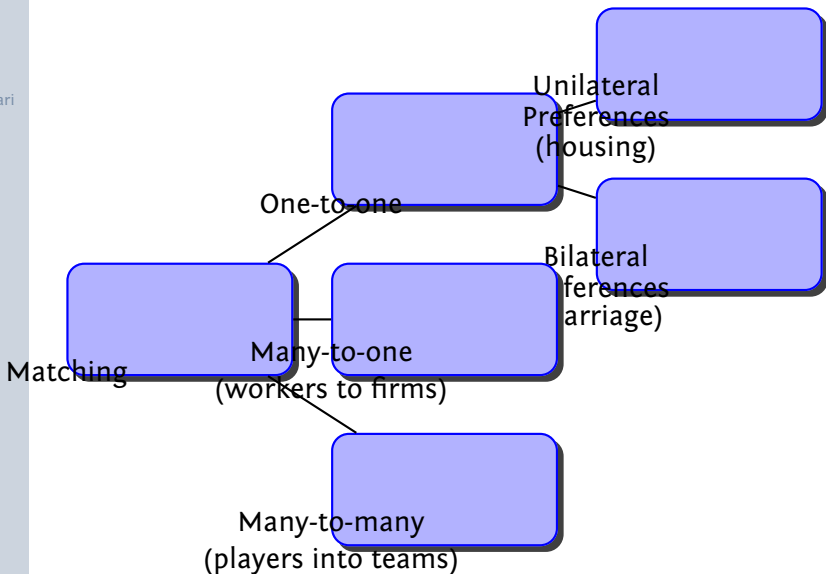
- Matching: payoffs depend on who matches with whom
- Examples:
 - Firm mergers
 - Firm upstream/downstream relationships
 - Workers and firms
 - Houses for consumers
 - Marriage
- Model primitive: payoffs of all potential matches
- Equilibrium: pairwise stability - no couple would prefer to deviate

Introduction

- Structural empirical matching models:
 - Data on observed matches and their characteristics
 - Goal: estimate payoff function

Types of matching

- Transferable vs non transferable utility



Matching – theory

- Much more developed than empirical work
- Optimal transportation theory – results imply existence and (in some cases) uniqueness of optimal matching; and existence, uniqueness, and efficiency of equilibrium
- See [Galichon \(2011\)](#) and references therein

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

Fox and Bajari
(2013)

References

Section 2

Sørensen (2007)

Sørensen (2007) “How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital”

- Fact: companies invested in by more experienced venture capitalists are more likely to go public

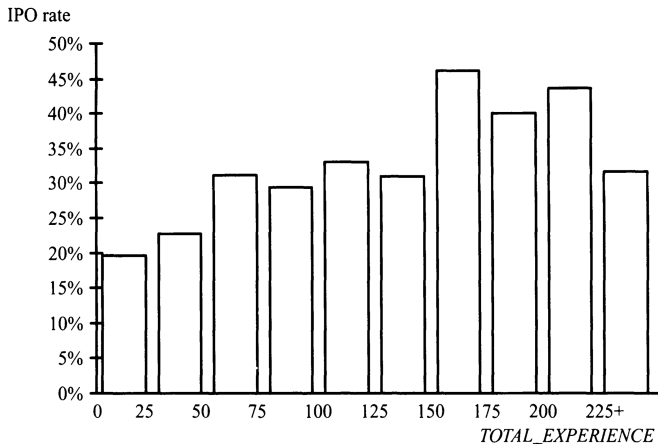


Figure 3. IPO rate in each group. The 1,666 companies are grouped into 10 groups according to the experience of the investor. The figure shows IPO rates of companies in each group.

Sørensen (2007) “How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital”

- Question: is this because experiences VCs invest in better companies or because experienced VCs' influence adds value to companies?
- Matching model used to distinguish these explanations

Why matching?

- VCs affect company value by:
 - Monitoring, management
 - Providing contacts
 - Signaling value to other investors
- Prior evidence that companies care about identity of investors; do not simply take best financial offer

- Set of investors I , set of companies J
- One-to-many: company has one investor; investor many companies
- Valuation of match V_{ij}
- Match correspondence μ
- Payoffs: non transferable
 - Investor: $\pi_i(\mu(i)) = \lambda \sum_{j \in \mu(i)} V_{ij}$
 - Company: $\pi_j = (1 - \lambda) V_{\mu(j)j}$
- Equilibrium: pairwise-stability
 - Opportunity cost of deviating for a pair that is not matched in μ

$$\bar{V}_{ij} \equiv V_{\mu(j)j} \vee \min_{j' \in \mu(i)} V_{ij'}$$

- Opportunity cost of remaining in match

$$\underline{V}_{ij} \equiv \max_{i' \in I: V_{i'j} > \min_{j' \in \mu(i')} V_{i'j'}} V_{i'j} \vee \min_{j' \in J: V_{ij'} > V_{ij'j'}} V_{ij'}$$

Model 2

- μ is stable $\Leftrightarrow V_{ij} < \bar{V}_{ij} \forall ij \notin \mu \Leftrightarrow V_{ij} > \bar{V}_{ij} \forall ij \in \mu$
- Define Γ_μ as set of all valuations such that μ is stable

Empirical model

- Observe μ , investor and company characteristics W_{ij} , X_{ij} , outcomes IPO_{ij}
- $V_{ij} = W'_{ij}\alpha + \eta_{ij}$
- Likelihood of matches: $P(\mu \in \Gamma_{\mu} - W\alpha)$
- Outcome: $IPO_{ij} = 1\{X_{ij}\beta + \epsilon_{ij} > 0\}$
- Assume $(\epsilon, \eta) \sim N$
- Estimate using MCMC

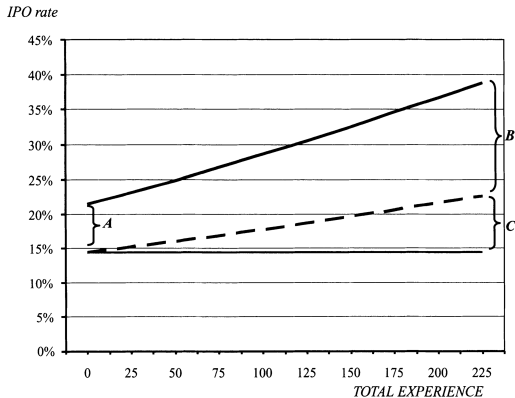


Figure 5. Decomposition of influence and sorting. The figure shows the IPO rates from the Probit model (Specification 1 in Table III) and the outcome equation of the structural model (Table V). The solid line indicates the IPO rate predicted by the Probit model. This rate is the empirical rate of the IPOs observed in the sample. The broken line represents the IPO rate from the outcome equation of the structural model. This is the IPO rate after controlling for the selection of the investments, and it represents the rate that would be observed if an average company were randomly assigned to investors with different degrees of experience.

Conclusions

- This paper: use a matching model to correct for selection
- Focus is not necessarily matching by itself
- Does not look at efficiency of matching or any counterfactuals related to matching
- Related work:
 - [Park \(2012\)](#): mutual fund mergers, very similar approach
 - [Uetake and Watanabe \(2012\)](#): bank mergers following deregulation, moment inequalities based on match stability

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

Fox and Bajari
(2013)

References

Section 3

Fox (2018)

Fox (2018) “Estimating matching games with transfers”

- Working paper version: [Fox \(2010a\)](#)
- Context: Car parts suppliers and automotive assemblers
- Goal: estimate revenues from producing different portfolios of parts
- Motivating examples:
 - GM considered divesting Opel – potential loss to suppliers who would not gain as much from specializing
 - Asian assembly plants enter North America – how beneficial to North American parts suppliers?

Fox (2018) “Estimating matching games with transfers”

- Data: part suppliers and assembler matches
 - No observed prices
- Approach: use equilibrium conditions of matching model to identify revenue function

- Two-sided, many-to-many
- Agent type $i \in I$ with measure $\eta(i)$
- Finite set of trades Ω , prices p_ω
- Valuation of i from buying Φ and selling Ψ

$$v^i(\Phi, \Psi) - \sum_{\omega \in \Phi} p_\omega + \sum_{\omega \in \Psi} p_\omega$$

Equilibrium

- Allocation $A : I \rightarrow \mathcal{P}(\Omega) \times \mathcal{P}(\Omega)$, with $A^i(\Phi, \Psi)$ = fraction of type i that buys Φ , and sells Ψ
- Competitive equilibrium is
 - Incentive compatible: $A^i(\Phi, \Psi) > 0$ only if

$$(\Phi, \Psi) \in \arg \max_{\tilde{\Phi}, \tilde{\Psi}} v^i(\tilde{\Phi}, \tilde{\Psi}) - \sum_{\omega \in \tilde{\Phi}} p_{\omega} + \sum_{\omega \in \tilde{\Psi}} p_{\omega}$$

- Feasible: for all $\omega \in \Omega$

$$\int_I \left(\sum_{\Phi: \omega \in \Phi} \sum_{\Psi} A^i(\Phi, \Psi) - \sum_{\Psi: \omega \in \Psi} \sum_{\Phi} A^i(\Phi, \Psi) d\eta(i) \right) = 0$$

- Equilibrium exists and is efficient **Azevedo and Hatfield (2018)**

Econometric Specification

- Observe: trades, part of agent type
- Don't observe: prices
- $i = (j, k)$, j observed, k unobserved

$$v^i(\Phi, \Psi) = \pi^j(\Phi, \Psi) + \epsilon_{\Phi, \Psi}^k$$

where ϵ^k is exchangeable for each j

- Parametric: $\pi^j(\Phi, \Psi) = X(j, \Phi, \Psi)' \theta$

Estimation

- Choice probability:

$$P_j(\Phi, \Psi) = \int_{\epsilon} \mathbf{1} \left[(\Phi, \Psi) \in \arg \max_{\Phi', \Psi'} X(j, \Phi', \Psi')' \theta + \epsilon_{\Phi', \Psi'}^k - \sum_{\Phi'} p_{\omega} + \sum_{\Psi'} p_{\omega} \right]$$

- Single agent rank order property:

$$P_j(\Phi_1, \Psi_1) \geq P_j(\Phi_2, \Psi_2) \text{ iff}$$

$$X(j, \Phi_1, \Psi_1)' \theta + - \sum_{\Phi_1} p_{\omega} + \sum_{\Psi_1} p_{\omega} \geq X(j, \Phi_2, \Psi_2)' \theta + - \sum_{\Phi_2} p_{\omega} + \sum_{\Psi_2} p_{\omega}$$

- Add together buyer and seller of a given ω to eliminate unobserved p_{ω} , add swapping that trade with a another buyer/seller trade to ensure feasibility

$$\begin{aligned} & X(b(\omega_1), \Phi_{b(\omega_1)}, \Psi_{b(\omega_1)})' \theta + X(s(\omega_1), \Phi_{s(\omega_1)}, \Psi_{s(\omega_1)})' \theta + \\ & X(b(\omega_2), \Phi_{b(\omega_2)}, \Psi_{b(\omega_2)})' \theta + X(s(\omega_2), \Phi_{s(\omega_2)}, \Psi_{s(\omega_2)})' \theta \\ & \geq \end{aligned}$$

$$\begin{aligned} & X(b(\omega_1), \bar{\Phi}_{b(\omega_1)}, \bar{\Psi}_{b(\omega_1)})' \theta + X(s(\omega_1), \bar{\Phi}_{s(\omega_1)}, \bar{\Psi}_{s(\omega_1)})' \theta + \\ & X(b(\omega_2), \bar{\Phi}_{b(\omega_2)}, \bar{\Psi}_{b(\omega_2)})' \theta + X(s(\omega_2), \bar{\Phi}_{s(\omega_2)}, \bar{\Psi}_{s(\omega_2)})' \theta \end{aligned}$$

- Maximum score: maximize sum of indicators of above inequalities to estimate θ

Empirical specification 1

- Data from SupplierBusiness on 941 suppliers, 11 assemblers with 46 brands, 260 models, and 34863 parts

- Data includes mainly North American and European firms

- Observation:

$\langle \underbrace{u}_{\text{part supplier}}, \underbrace{d}_{\text{car model}}, \underbrace{l}_{\text{part}} \rangle$ in market
 Federal-Mogul Fiat front pads
 $\underbrace{h}_{\text{component}}$
 disk brakes

- Variables in valuations:
 - $X(j^s, \Psi)$ = measure of specialization of supplier in its matches, specifically HHI of parts across

Empirical specification 2

- Continents, assemblers, brands, models
- $\chi(j^b, \Phi)$ = measure of specialization of parts in its matches, specifically HHI of suppliers across
 - Assemblers, brands, models

TABLE 1. Specialization by Suppliers and Assemblers

HHI Measure	Valuation Function Estimates		Sample Statistics for HHI Measures	
	Point Estimate	95% CI Set Identified	Mean	Standard Deviation
		Suppliers		
Parent Group	+1	Superconsistent	0.35	0.28
Continent	1.04	(0.0482, 9.45)	0.76	0.18
Brand	23.9	(1.29, 121)	0.25	0.27
Model	376	(278, 933)	0.17	0.26
		Assemblers		
Parent Group	-0.007	(-1.30, 0.202)	0.14	0.11
Brand	-0.005	(-1.99, 0.705)	0.35	0.33
Model	-0.003	(-3.36, 33.5)	0.58	0.60
# Inequalities	298,272			
% Satisfied	82.3%			

Note: The parameter on parent group specialization is fixed at +1. Estimating it with a smaller number of inequalities always finds the point estimate of +1, instead of -1. The estimate of a parameter that can take only two values is superconsistent, so I do not report a confidence interval. See Online Appendix B for details on estimation and inference.

Results

TABLE 2. Percentage Valuation Change by Suppliers From GM Divesting Opel

Quantile	
0	-0.0032
0.10	-0.0014
0.25	-0.0008
0.50 (median)	-0.0004
0.75	-0.0002
0.90	-0.00008
1	~0

Note: This table uses the point estimates from Table 1 to calculate the valuations from observable types of suppliers before and after GM divests Opel. In the model, Opel becomes a separate parent group. For each supplier, selling one or more parts to Opel and one or more cars to another GM brand, I calculate $\frac{\theta_{PG}^S \Delta X_{PG}(j^S, \Psi^S)}{\bar{X}(j^S, \Psi^S) \theta}$. Each supplier that operates in multiple component categories (markets) is treated separately in each component category.

TABLE 3. Supplier Competitive Advantages From Asian Assemblers

HHI Measure	Valuation Function Estimates	
	Point Estimate	95% CI
	Suppliers	
Parent Group	+1	Superconsistent
Continent	1.03	(0.045, 13.7)
Brand	24.2	(1.09, 235)
Model	388	(363, 898)
Competitive Advantage	-0.261	(-30.0, 32.2)
	Assemblers	
Parent Group	-0.0101	(-1.50, 0.224)
Brand	-0.00789	(-2.07, 0.831)
Model	-0.00437	(-3.64, 34.2)
# Inequalities		298,272
% Satisfied		82.3%

Note: The parameter on parent group specialization is fixed at +1. Estimating it with a smaller number of inequalities always finds the point estimate of +1, instead of -1. The estimate of a parameter that can take only two values is superconsistent, so I do not report a confidence interval. See Online Appendix B for details on estimation and inference.

Other applications and extensions 1

- Identification: Fox (2010b)
- Unobserved heterogeneity: Fox, Yang, and Hsu (2018)
- Matching maximum score estimator of Fox (2018) used in
 - Fox and Bajari (2013): FCC spectrum auction – no trades after auction implies pairwise stability
 - Oktay Akkus and Hortaçsu (2012): bank mergers, matching with observed transfers
 - Levine (2009): pharmaceutical marketing firms and drugs

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

**Fox and Bajari
(2013)**

References

Section 4

Fox and Bajari (2013)

Fox and Bajari (2013) “Measuring the Efficiency of an FCC Spectrum Auction”

- Estimates an auction model using pairwise stability
- C block FCC spectrum auctions 1995-1996
 - Simultaneous ascending auctions for 480 geographic areas
 - Theory & evidence from other FCC auctions suggests collusion
- Goal: estimate distribution of valuations & and allocative efficiency
- Identifying assumption: allocation of licenses is pairwise stable in matches, that is, an exchange of two licenses by winning bidders must not raise the sum of the valuations of the two bidders

Background

- C block 1900 MHz spectrum used for mobile phones
- Auction format:
 - Multiple rounds
 - Each round, simultaneously submit bids (or not) on all 480 regions
 - Auction ends when no more bids placed on any item in a round
 - Lasted 185 days
- Only new carriers participated (small business discount)
- 255 bidders, 85 winners, most either went bankrupt or merged with incumbent carriers

Matching

Paul Schrimpf

Introduction

Sørensen
(2007)

Fox (2018)

Fox and Bajari
(2013)

References

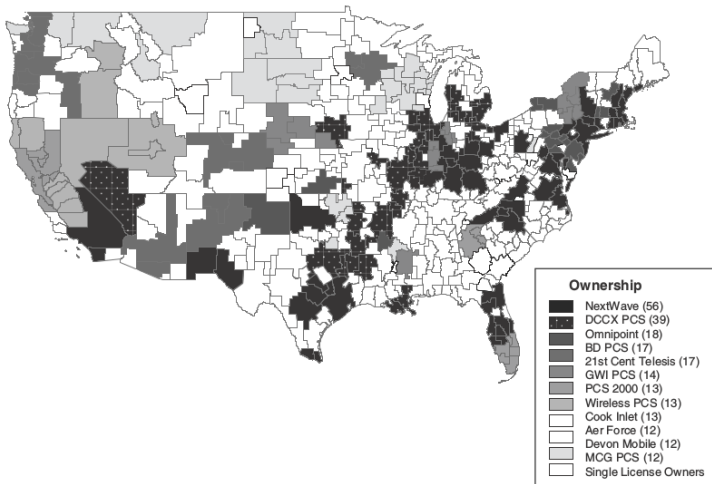


FIGURE 1. MAP OF THE LICENSES WON BY THE TOP 12 WINNING BIDDERS AND BIDDERS WHO WON ONLY ONE LICENSE

Suggestive evidence of intimidatory collusion

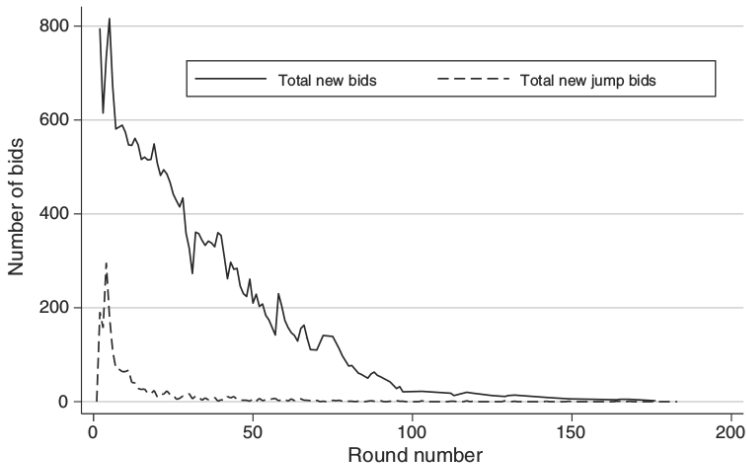


FIGURE 3. THE NUMBER OF JUMP BIDS PER ROUND

Valuations

- $a = 1, \dots, N$ bidders, $j = 1, \dots, L$ licenses
- Profit of bidder a from $J \subset L$

$$\pi_a(J) = \sum_{j \in J} p_j$$

- Parameterization:

$$\pi_a(J) = \underbrace{\bar{\pi}_\beta(\mathbf{w}_a, \mathbf{x}_J)}_{\pm 1 \cdot \text{elig}_a \cdot (\sum_{j \in J} \text{pop}_j) + \beta' \text{complem}_j} + \sum_{j \in J} \xi_j + \sum_{j \in J} \epsilon_{a,j}$$

- $\bar{\pi}(\mathbf{w}, \mathbf{x})$ and ξ_j common knowledge of bidders, ξ_j unobserved by econometrician
- ϵ i.i.d., private for bidders, unobserved by econometrician

Measuring complementarities

TABLE 2—WINNING PACKAGES: SAMPLE STATISTICS AND CORRELATION MATRIX FOR GEOGRAPHIC COMPLEMENTARITY PROXIES

Characteristic	Mean	SD	Min	Max
Population/distance two markets in a package	0.0055	0.024	0	0.20
Trips between markets in a package in the American Travel Survey	0.0032	0.020	0	0.182
Total trips between airports in markets in a package (thousands)	0.0023	0.017	0	0.150
Correlations	Geo distribution		ATS trips	
Population/distance two markets in a package	1			
Trips between markets in a package in the American Travel Survey	0.97		1	
Total trips between airports in markets in a package (thousands)	0.95		0.99	

Notes: The sample is the 85 winning packages in the continental United States. The formulas for these measures are equations (3) and (4).

- Each bidder makes a payment before the auction begins for initial eligibility. A bidder's eligibility is expressed in units of total population. A bidder cannot bid on a package of licenses that exceeds the bidder's eligibility.

$$\bullet \text{ geomcomplem}_j = \sum_{i \in J} \text{pop}_i \frac{\sum_{j \in L \setminus \{i\}} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^\delta}}{\sum_{j \in L \setminus \{i\}} \frac{\text{pop}_i \text{pop}_j}{\text{dist}_{i,j}^\delta}}$$

$$\bullet \text{ travelcomplem}_j = \sum_{i \in J} \text{pop}_i \frac{\sum_{j \in L \setminus \{i\}} \text{trips}(\text{origin } i, \text{destination } j)}{\sum_{j \in L \setminus \{i\}} \text{trips}(\text{origin } i, \text{destination } j)}$$

Pairwise stability

- Pairwise stable in matches:

$$\pi_a(U_a) + \pi_b(U_b) \geq \pi_a((U_a \setminus \{i_a\}) \cup \{i_b\}) + \pi_b((U_b \setminus \{i_b\}) \cup \{i_a\})$$

- Evidence for:
 - Often satisfied in experimental data
 - Swaps did not occur after auction
 - Holds in theoretical models of ascending auctions with collusion: Brusco & Lopomo (2002) and Engelbrecht-Wiggans & Kahn (2005) and Milgrom (200)

Estimation

- Similar to Fox (2010a), but without transfers
- Objective function = sum of indicators for pairwise stability inequalities
- Fixed effects drop out of pairwise stability conditions from differencing
- Inference through subsampling

TABLE 3—MAXIMUM RANK CORRELATION ESTIMATES OF VALUATION PARAMETERS

Column	(1)	(2)	(3)	(4)
Distance parameter δ		4		2
Population \times bidder eligibility	+1	+1	+1	+1
		Superconsistent		
Population/distance two markets in a package	0.32 (0.31, 0.50)	0.32 (0.30, 0.47)	1.06 (0.87, 1.56)	0.86 (0.58, 1.06)
Trips between markets in a package in the American Travel Survey		0.03 (-0.08, 0.40)		-0.62 (-0.96, -0.27)
Total trips between airports in markets in a package (thousands)		-0.16 (-0.37, 0.34)		-0.26 (-0.51, 0.51)
Number possible inequalities		13,428		
Percent inequalities correct	0.944	0.945	0.956	0.960

Notes: The objective function was numerically maximized using differential evolution (Storn and Price 1997). More than ten runs were performed for all specifications. The reported point estimates are the best found maxima. The parentheses are 95 percent confidence intervals computed using subsampling. Subsampling uses 200 replications and 25 packages per replication (sampled without replacement). For each 25 packages, we use only the inequalities where all licenses are from the sampled packages. Subsampled confidence regions are not necessarily symmetric around the point estimate. In unreported results, we take subsets of the data by using only the inequalities corresponding to 120 out of the 480 licenses in the United States. For each license, we evaluate the valuation functions using the full winning package, whether all of the package's licenses are among the subset of 120 or not. The confidence regions from drawing licenses are similar to the regions found by drawing packages. Subsampling has not been extended to allow for spatial autocorrelation, so we do not adjust for such correlation. Parameters that can take on only a finite number of values (here ± 1) converge at an arbitrarily fast rate; they are superconsistent.

Interpretation

- SD of $elig \cdot (\sum pop)$ 0.029, SD of $geocomplem$ is 0.024
- $\beta_{geo} = 0.32$, so $geocomplem$ 32% as important as population

TABLE 4—ESTIMATORS USING OTHER INEQUALITIES

Type of inequalities	Transfer of 1 license		Swaps of 1 license w/prices	
	(1)	(2)	(3)	(4)
Population × bidder eligibility	+1	+1	0.36	0.36
	Superconsistent		(−0.13, 0.41)	(−0.15, 0.42)
Population/distance two markets in a package	6.7	9.8	0.12	0.12
	(−3.0, 9.2)	(−12, 14)	(−0.23, 0.15)	(−4.82, 0.15)
Trips between markets in a package in the American Travel Survey		−0.37		0.03
		(−0.49, 1.2)		(−0.81, 0.19)
Total trips between airports in markets in a package (thousands)		−0.1		−0.09
		(−0.39, 0.06)		(−0.22, 0.04)
Price (in trillions)			−1	−1
			Superconsistent	
Number possible inequalities	16,084		73,409	
Percent inequalities correct	0.950	0.953	0.913	0.914

Notes: All estimates use $\delta = 4$. See Table 3 for computational details.

- 6.7 and 9.8 are implausibly large – imply increasing complementarity is worth as much as having 6 times the population in the area
- Model with prices implies value of nationwide license is \$360 billion, but total bids were \$10 billion, annual revenues in 2006 were \$113 billion

TABLE 5—COUNTERFACTUAL DETERMINISTIC EFFICIENCY FROM FIVE ALLOCATIONS:
POINT ESTIMATES IMPOSING ELIGIBILITY CONSTRAINTS

Allocation	$\text{elig}_d(\sum_{j \in J} \text{pop}_j)$	Geographic distance	Air travel	ATS trips	Total
C block: 85 winning packages	$1 \cdot 0.39 =$ 0.39	$0.32 \cdot 0.47 =$ 0.15	$-0.16 \cdot 0.20 =$ -0.03	$0.03 \cdot 0.27 =$ 0.01	0.52
All 480 licenses won by different bidders	$1 \cdot 0.17 =$ 0.17	$0.32 \cdot 0 =$ 0	$-0.16 \cdot 0 =$ 0	$0.03 \cdot 0 =$ 0	0.17
Each 47 MTAs separate package	$1 \cdot 0.20 =$ 0.20	$0.32 \cdot 0.72 =$ 0.23	$-0.16 \cdot 0.04 =$ -0.01	$0.03 \cdot 0.17 =$ 0	0.43
Four large, regional licenses (top four of the 85 actual winners win)	$1 \cdot 0.50 =$ 0.50	$0.32 \cdot 0.96 =$ 0.31	$-0.16 \cdot 0.37 =$ -0.06	$0.03 \cdot 0.58 =$ 0.02	0.77
Nationwide license for entire United States (NextWave wins)	$1 \cdot 0.71 =$ 0.71	$0.32 \cdot 1 =$ 0.32	$-0.16 \cdot 1 =$ -0.16	$0.03 \cdot 1 =$ 0.03	0.90

Notes: Eligibility, population, and all three complementarity proxies range from 0 to 1. These counterfactuals use the point estimates from column 2 of Table 3. Only licenses in the continental United States are considered. For the 47 MTAs in the continental United States, as well as the four large regions, the top winners in the actual auction are assortatively matched to the counterfactual packages in order of population. For example, NextWave always wins the package with the highest population.

Agarwal, Nikhil. 2012. “An Empirical Model of the Medical Match.” URL <http://www.people.fas.harvard.edu/~agarwal3/papers/AgarwalJMP.pdf>.

Azevedo, Eduardo M and John William Hatfield. 2018. “Existence of equilibrium in large matching markets with complementarities.” URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3268884.

Choo, Eugene and Aloysius Siow. 2006. “Who Marries Whom and Why.” *Journal of Political Economy* 114 (1):pp. 175–201. URL <http://www.jstor.org/stable/10.1086/498585>.

Fox, Jeremy T. 2009. “Matching models: empirics.” In *The New Palgrave Dictionary of Economics*, edited by Steven N. Durlauf and Lawrence E. Blume. Basingstoke: Palgrave Macmillan. URL http://www.dictionaryofeconomics.com/article?id=pde2009_M000426.

- . 2010a. “Estimating Matching Games with Transfers.” Working paper, University of Michigan. URL http://www-personal.umich.edu/~jtfox/working-papers/matching_fox.pdf.
- . 2010b. “Identification in matching games.” *Quantitative Economics* 1 (2):203–254. URL <http://dx.doi.org/10.3982/QE3>.
- . 2018. “Estimating matching games with transfers.” *Quantitative Economics* 9 (1):1–38. URL <https://onlinelibrary.wiley.com/doi/abs/10.3982/QE823>.
- Fox, Jeremy T. and Patrick Bajari. 2013. “Measuring the Efficiency of an FCC Spectrum Auction.” *American Economic Journal: Microeconomics* 5 (1):100–146. URL <http://www.aeaweb.org/articles.php?doi=10.1257/mic.5.1.100>.

- Fox, Jeremy T., Chenyu Yang, and David H. Hsu. 2018. "Unobserved Heterogeneity in Matching Games." *Journal of Political Economy* 126 (4):1339–1373. URL <https://doi.org/10.1086/697740>.
- Galichon, Alfred. 2011. "Theoretical and Empirical Aspects of Matching Markets." URL <http://alfredgalichon.com/wp-content/uploads/2012/10/MatchingnotesColumbia.pdf>.
- Galichon, Alfred and Bernard Salanié. 2010. "Matching with trade-offs: revealed preferences over competing characteristics." URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1640380.
- Graham, B. 2011. "Econometric methods for the analysis of assignment problems in the presence of complementarity and social spillovers." *Handbook of social economics* 1:965–1052. URL http://128.32.105.3/~bgraham/Published/HandbookOfSocialEconomics_1B_2011/BSG_HandbookOfSocialEconomics_1B_2011.pdf.

Levine, Anna A. 2009. "Licensing and scale economies in the biotechnology pharmaceutical industry." URL <http://apps.olin.wustl.edu/workingpapers/pdf/2010-11-004.pdf>.

Oktay Akkus, J. Anthony Cookson and Ali Hortaçsu. 2012. "The Determinants of Bank Mergers: A Revealed Preference Analysis." URL <https://docs.google.com/uc?export=download&id=0B9xl-vM6J0oDMjVtVXhiNWt5bjA>.

Park, Minjung. 2012. "Understanding Merger Incentives and Outcomes in the US Mutual Fund Industry." URL <http://faculty.haas.berkeley.edu/mpark/MergerPaper.pdf>.

Sørensen, Morten. 2007. "How Smart Is Smart Money? A Two-Sided Matching Model of Venture Capital." *The Journal of Finance* 62 (6):pp. 2725–2762. URL <http://www.jstor.org/stable/4622352>.

Uetake, Kosuke and Yasutora Watanabe. 2012. “Entry by Merger: Estimates from.” URL <http://www.kellogg.northwestern.edu/faculty/watanabe/htm/EntryByMerger.pdf>.