Demand and supply of differentiated products

Paul Schrimpf

UBC
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Part I

Implementation

Computational issues

- Non-convex optimization problems are almost always difficult to solve, this is no exception
- Nested iteration can be problematic
  - Solve for $\delta(theta)$:

    ```
    while norm(T(delta) - delta) > tolerance1 { delta
    ```
  - Minimize

    ```
    while norm(theta - thetaOld) > tolTheta && norm(f
      thetaOld = theta
      fold = f
      // update theta by e.g. newton's method, set f =
    }
    ```
  - Error in δ can lead to error in minimization
  - Error in δ is not a continuous with respect to θ (where changing θ changes number of iterations)

Nevo

- Popular code provided by Nevo (2000)
- Requires: Matlab, optimization toolbox
- Nevo's code does not run in current version of Matlab, but Rasmusen (2006) update does
- Code runs in Octave after changing `fminsearch` to another optimization routine
- Worked on by three people
- Used by at least six other papers (see Knittel and Metaxoglou (2014) footnote 5 for list)
- Fast for data provided

Nevo - issues

- Minimization difficult and not robust
  - Starting value
  - Algorithm
  - Tolerance for finding δ (Dubé, Fox, and Su (2012b) show loose tolerance affects estimates)
  - Knittel and Metaxoglou (2014) algorithms often stop at point where first and/or second order conditions fail
  - Knittel and Metaxoglou (2014) differences among convergence points economically significant

Dubé, Fox, and Su (2012b) 1

- Fixed point iteration to compute δ messes up GMM minimization; also is not best method for finding δ
    - Table 1: shows problem is too large a tolerance. NFP gives good estimates when tolerances are tight
- Can recast problem as constrained minimization

$$\min_{\theta,\delta} \sum_{\ell} \left(\frac{1}{JT} \sum_{j=1}^{J} \sum_{t=1}^{T} \xi_{jt}(\theta, \delta) f_{\ell}(w_t) \right)^2$$

subject to

$$\hat{s} = \sigma(\cdot; \theta, \delta)$$

- Su and Judd (2012): "mathematical programming with equilibrium constraints" (MPEC)
- Use state of the art algorithm to solve constrained minimization

Dubé, Fox, and Su (2012b) 2

- Solvers work best with accurate (i.e. not finite difference) derivatives—supplying 1st and 2nd order derivatives makes algorithm take approximately 1/3 as long as with just 1st order (Dubé, Fox, and Su, 2012a)
- Gains from exploiting sparsity of Jacobian of constraints and Hessian of objective function

Dubé, Fox, and Su (2012b) code

- Code requires: Matlab, KNITRO
- KNITRO proprietary, free version limited to 300 variables & constraints
- KNITRO can be replaced with other optimization algorithm, but others do not seem to work as well:
    - IPOPT uses similar algorithm, but I had trouble installing
    - NLOPT has no interior point algorithm, its algorithms do not seem to deal with nonlinear constraints very well
    - Skrainka (2012) uses SNOPT, which is similar algorithm to NLOPT's SLSQP
- Runs in Octave with KNITRO replaced by NLOPT

Observations

- High quality commercial solver appears necessary; my attempts with NLOPT fail and take longer
  - Skrainka (2012) uses SNOPT instead of KNITRO
- KNITRO and SNOPT not perfect
  - Still sensitive to starting values
  - MPEC replaces a contraction – a problem we know we can solve – with constraints that may make the optimization harder
    - Reynaerts, Varadhan, and Nash (2012) give method to improve accuracy and speed of computing δ
    - Dubé, Fox, and Su (2012a) using nested fixed point requires fewer solver iterations than MPEC, but takes as long or longer because of time spend solving for δ (can be much longer if contraction mapping is slow)
- Reynaert and Verboven (2014): using optimal instruments makes optimization more robust

References about implementation

- Overviews
  - Nevo (2000)
  - Dubé, Fox, and Su (2012b), Dubé, Fox, and Su (2012a)
  - Knittel and Metaxoglou (2014)
  - Skrainka (2012)
- Particular issues
  - Skrainka and Judd (2011): integration
  - Reynaerts, Varadhan, and Nash (2012): solving for δ
- Course on discrete choice models with simulation by Kenneth Train `http://elsa.berkeley.edu/users/train/distant.html`
- Bayesian: Jiang, Manchanda, and Rossi (2009), Brian Viard, Gron, and Polson (2014), Sun and Ishihara (2013)
- Overview of optimization methods and software Leyffer and Mahajan (2010)

Section 1

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