

Estimating Production Functions

Introduction

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Economics 567

January 11, 2023

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Section 1

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Why estimate production functions?

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- Primitive component of economic model
- Gives estimate of firm productivity – useful for understanding economic growth
 - Stylized facts to inform theory, e.g. Foster, Haltiwanger, and Krizan (2001)
 - Effect of deregulation, e.g. Olley and Pakes (1996)
 - Growth within old firms vs from entry of new firms, e.g. Foster, Haltiwanger, and Krizan (2006)
 - Effect of trade liberalization, e.g. Amiti and Konings (2007)
 - Effect of FDI Javorcik (2004)

General references:

- **Aguirregabiria (2021) chapter 3**
- De Loecker and Syverson (2021)
- Ackerberg et al. (2007) section 2
- Van Beveren (2012)

Section 2

Setup

- Cobb Douglas production

$$Y_{it} = A_{it} K_{it}^{\beta_k} L_{it}^{\beta_\ell}$$

- In logs,

$$y_{it} = \beta_k k_{it} + \beta_\ell l_{it} + \omega_{it} + \epsilon_{it}$$

with $\log A_{it} = \omega_{it} + \epsilon_{it}$, ω_{it} known to firm, ϵ_{it} not

Empirical Challenges

- 1 Simultaneity: if firm has information about $\log A_{it}$ when choosing inputs, then inputs correlated with $\log A_{it}$
- 2 Selection: firms with low productivity will exit sooner
- 3 Others: measurement error, specification

Section 3

Simultaneity

Simultaneity

- Firm's inputs will be correlated with firm's knowledge of productivity
- E.g. output price p , wage w , choosing L given K

$$\max_L pE[A]K_k^\beta L_\ell^\beta - wL$$

implies

$$L = \left(\frac{p}{w} \beta_\ell E[A] K_k^{\beta_k} \right)^{\frac{1}{1-\beta_\ell}}$$

or in logs,

$$l = \frac{1}{1-\beta_\ell} \log \left(\frac{p}{w} \beta_\ell \right) + \frac{\beta_k}{1-\beta_\ell} k + \frac{1}{1-\beta_\ell} \log (E[A])$$

- l correlated with productivity through correlation of $\log A$ and $\log(E[A])$
- Exercise: calculate bias of $\hat{\beta}_\ell^{OLS}$ (with some further assumptions)

Simultaneity solutions

- 1 IV
- 2 Panel data
- 3 Control functions

Instrumental variables

- Instrument must be
 - Correlated with k and l
 - Uncorrelated with $\omega + \epsilon$
- Possible instrument: input prices
 - Correlated with k, l through first-order condition
 - Uncorrelated with ω if input market competitive
- Other possible instruments: output prices (more often endogenous), input supply or output demand shifter (hard to find)

Problems with input prices as IV

- Not available in many data sets
- Average input price of firm could reflect quality as well as price differences
- Need variation across observations
 - If firms use homogeneous inputs, and operate in the same output and input markets, we should not expect to find any significant cross-sectional variation in input prices
 - If firms have different input markets, maybe variation in input prices, but different prices could be due to different average productivity across input markets
 - Variation across time is potentially endogenous because could be driven by time series variation in average productivity

Fixed effects

- Have panel data, so should consider fixed effects
- FE consistent if:
 - 1 $\omega_{it} = \eta_i + \delta_t + \omega_{it}^*$
 - 2 ω_{it}^* uncorrelated with l_{it} and k_{it} , e.g. ω_{it}^* only known to firm after choosing inputs
 - 3 ω_{it}^* not serially correlated and is strictly exogenous
- Problems:
 - Fixed productivity a strong assumption
 - Estimates often small in practice
 - Worsens measurement error problems

$$\text{Bias}(\hat{\beta}_k^{FE}) \approx -\frac{\beta_k \text{Var}(\Delta\epsilon)}{\text{Var}(\Delta k) + \text{Var}(\Delta\epsilon)}$$

Dynamic panel: motivation 1

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- General idea: relax fixed effects assumption, but still exploit panel
- Collinearity problem: Cobb-Douglas production, flexible labor and capital implies log labor and log capital are linear functions of prices and productivity (**Bond and Söderbom (2005)**)
- If observed labor and capital are not collinear then there must be something unobserved that varies across firms (e.g. prices), but that could invalidate monotonicity assumption of control function

Dynamic panel: moment conditions

- See Blundell and Bond (2000)
- Assume $\omega_{it} = \gamma_t + \eta_i + v_{it}$ with $v_{it} = \rho v_{i,t-1} + e_{it}$, so

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \gamma_t + \eta_i + v_{it} + \epsilon_{it}$$

subtract $\rho y_{i,t-1}$ and rearrange to get

$$y_{it} = \rho y_{i,t-1} + \beta_l (l_{it} - \rho l_{i,t-1}) + \beta_k (k_{it} - \rho k_{i,t-1}) + \gamma_t - \rho \gamma_{t-1} + \underbrace{\eta_i(1 - \rho)}_{=\eta_i^*} + \underbrace{e_{it} + \epsilon_{it} - \rho \epsilon_{i,t-1}}_{=w_{it}}$$

- Moment conditions:
 - Difference: $E[x_{i,t-s} \Delta w_{it}] = 0$ where $x = (l, k, y)$
 - Level: $E[\Delta x_{i,t-s} (\eta_i^* + w_{it})] = 0$

Dynamic panel: economic model 1

- Adjustment costs

$$V(K_{t-1}, L_{t-1}) = \max_{I_t, K_t, H_t, L_t} P_t F_t(K_t, L_t) - P_t^K (I_t + G_t(I_t, K_{t-1})) - W_t (L_t + C_t(H_t, L_{t-1})) + \psi E [V(K_t, L_t) | \mathcal{I}_t]$$

$$\text{s.t. } K_t = (1 - \delta_k) K_{t-1} + I_t$$

$$L_t = (1 - \delta_l) L_{t-1} + H_t$$

Implies

$$P_t \frac{\partial F_t}{\partial L_t} - W_t \frac{\partial C_t}{\partial L_t} = W_t + \lambda_t^L \left(1 - (1 - \delta_l) \psi E \left[\frac{\lambda_{t+1}^L}{\lambda_t^L} | \mathcal{I}_t \right] \right)$$

$$P_t \frac{\partial F_t}{\partial K_t} - P_t^K \frac{\partial G_t}{\partial K_t} = \lambda_t^K \left(1 - (1 - \delta_k) \psi E \left[\frac{\lambda_{t+1}^K}{\lambda_t^K} | \mathcal{I}_t \right] \right)$$

Dynamic panel: economic model 2

- Current productivity shifts $\frac{\partial F_t}{\partial L_t}$ and (if correlated with future) the shadow value of future labor $E \left[\frac{\lambda_{t+1}^L}{\lambda_t^L} | \mathcal{I}_t \right]$
- Past labor correlated with current because of adjustment costs

Dynamic panel data: problems

- Problems:
 - Sometimes imprecise (especially if only use difference moment conditions)
 - Differencing worsens measurement error
 - Weak instrument issues if only use difference moment conditions but levels stronger (see [Blundell and Bond \(2000\)](#))
 - Level moments require stronger stationarity assumption
 - η_j uncorrelated with Δx_{it}

Control functions

- From **Olley and Pakes (1996)** (OP)
- **Control function:** function of data conditional on which endogeneity problem solved
 - E.g. usual 2SLS $y = x\beta + \epsilon$, $x = z\pi + v$, control function is to estimate residual of reduced form, \hat{v} and then regress y on x and \hat{v} . \hat{v} is the control function
- Main idea: model choice of inputs to find a control function

OP assumptions

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it}$$

- 1 ω_{it} follows exogenous first order Markov process,

$$p(\omega_{it+1} | \mathcal{I}_{it}) = p(\omega_{it+1} | \omega_{it})$$

- 2 Capital at t determined by investment at time $t - 1$,

$$k_t = (1 - \delta)k_{it-1} + i_{it-1}$$

- 3 Investment is a function of ω and other observed variables

$$i_{it} = I_t(k_{it}, \omega_{it}),$$

and is strictly increasing in ω_{it}

- 4 Labor variable and non-dynamic, i.e. chosen each t , current choice has no effect on future (can be relaxed)

OP estimation of β_ℓ

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- Invertible investment implies $\omega_{it} = I_t^{-1}(k_{it}, i_{it})$

$$\begin{aligned} y_{it} &= \beta_k k_{it} + \beta_\ell l_{it} + I_t^{-1}(k_{it}, l_{it}) + \epsilon_{it} \\ &= \beta_\ell l_{it} + f_t(k_{it}, i_{it}) + \epsilon_{it} \end{aligned}$$

- Partially linear model
 - Estimate by e.g. regress y_{it} on l_{it} and series functions of t, k_{it}, i_{it}
 - Gives $\hat{\beta}_\ell, \hat{f}_{it} = \hat{f}_t(k_{it}, i_{it})$

- Note: $\hat{f}_t(k_{it}, i_{it}) = \hat{\omega}_{it} + \beta_k k_{it}$
- By assumptions, $\omega_{it} = E[\omega_{it} | \omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$ with $E[\xi_{it} | k_{it}] = 0$
- Use $E[\xi_{it} | k_{it}] = 0$ as moment to estimate β_k .
 - OP: write production function as

$$\begin{aligned} y_{it} - \beta_l l_{it} &= \beta_k k_{it} + g(\omega_{it-1}) + \xi_{it} + \epsilon_{it} \\ &= \beta_k k_{it} + g(f_{it-1} - \beta_k k_{it-1}) + \\ &\quad + \xi_{it} + \epsilon_{it} \end{aligned}$$

Use $\hat{\beta}_l$ and \hat{f}_{it} in equation above and estimate $\hat{\beta}_k$ by e.g. semi-parametric nonlinear least squares

- **Ackerberg, Caves, and Frazer (2015)**: use $E[\hat{\xi}_{it}(\beta_k)k_{it}] = 0$

Dynamic panel vs control function

- Both derive moment conditions from assumptions about timing and information set of firm
- Dealing with ω
 - Dynamic panel: AR(1) assumption allows quasi-differencing
 - Control function: makes ω estimable function of observables
- Dynamic panel allows fixed effects, does not make assumptions about input demand
- Control function allows more flexible process for ω_{it}

Applications

- Olley and Pakes (1996): productivity in telecom after deregulation
- Söderbom, Teal, and Harding (2006): productivity and exit of African manufacturing firms, uses IV
- Levinsohn and Petrin (2003): compare estimation methods using Chilean data
- Javorcik (2004): FDI and productivity, uses OP
- Amiti and Konings (2007): trade liberalization in Indonesia, uses OP
- Aw, Chen, and Roberts (2001): productivity differentials and firm turnover in Taiwan
- Kortum and Lerner (2000): venture capital and innovation

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Selection

Selection

- Let $d_{it} = 1$ if firm in sample.
 - Standard conditions imply $d = 1\{\omega \geq \omega^*(k)\}$
- Messes up moment conditions
 - All estimators based on $E[\omega_{it}\text{Something}] = 0$, observed data really use $E[\omega_{it}\text{Something}|d_{it} = 1]$
 - E.g. OLS okay if $E[\omega_{it}|l_{it}, k_{it}] = 0$, but even then,

$$\begin{aligned} E[\omega_{it}|l_{it}, k_{it}, d_{it} = 1] &= E[\omega_{it}|l_{it}, k_{it}, \omega_{it} \geq \omega^*(k_{it})] \\ &= \lambda(k_{it}) \neq 0 \end{aligned}$$

- Selection bias negative, larger for capital than labor

- Estimate β_ℓ as above
- Write

$$d_{it} = 1\{\xi_{it} \leq \omega^*(k_{it}) - \rho(f_{i,t-1} - \beta_k k_{it-1}) = h(k_{it}, f_{it-1}, k_{it-1})\}$$
- Propensity score $P_{it} \equiv E[d_{it}|k_{it}, f_{it-1}, k_{it-1}]$
- Similar to before estimate β_k , from

$$y_{it} - \beta_\ell l_{it} = \beta_k k_{it} + \tilde{g}(f_{it-1} - \beta_k k_{it-1}, P_{it}) + \xi_{it} + \epsilon_{it}$$

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